Application of Derivatives in Business and Economics

# Elasticity of Demand

In calculus, the derivative represents the rate of change of a function with respect to its independent variable. In the case of elasticity of demand, we are interested in the rate of change of quantity demanded with respect to price.

By taking the derivative of the demand function with respect to price, we can determine how sensitive the quantity demanded is to changes in price. This sensitivity is precisely what elasticity of demand measures.

The formula for elasticity of demand is:

E = (dQ/dP) \* (P/Q)

Here, (dQ/dP) represents the derivative of the demand function (Q) with respect to price (P). It tells us how the quantity demanded changes for a small change in price.

The derivative (dQ/dP) is multiplied by the ratio of price to quantity demanded (P/Q). This ratio captures the proportional relationship between price and quantity demanded.

When we calculate the derivative and substitute the relevant values, we obtain the elasticity of demand. The sign and magnitude of this elasticity tell us whether demand is elastic, inelastic, or unitary.

By utilizing calculus and derivatives, we can not only determine the elasticity of demand at a specific price point but also analyze how changes in price affect the quantity demanded. This understanding is crucial for businesses and policymakers in making pricing decisions, optimizing revenue, and predicting market behavior.

## EXAMPLE

Suppose we have a demand function for a product given by:

Q = 100 - 2P

where Q represents the quantity demanded and P represents the price.

To find the elasticity of demand, we need to take the derivative of the demand function with respect to price (P):

dQ/dP = -2

Now, let's consider a specific scenario where the price is $10. We can substitute this value into the demand function to find the corresponding quantity demanded:

Q = 100 - 2(10)

Q = 100 - 20

Q = 80

So, at a price of $10, the quantity demanded is 80 units.

Next, we can calculate the elasticity of demand at this price point by using the formula:

E = (dQ/dP) \* (P/Q)

Plugging in the values we obtained earlier:

E = (-2) \* (10/80)

E = -0.25

In this example, the elasticity of demand at a price of $10 is -0.25. The negative sign indicates that demand is elastic.

An elasticity of demand of -0.25 means that a 1% decrease in price would lead to a 0.25% increase in quantity demanded. The quantity demanded is relatively responsive to price changes.

# Revenue Functions

Revenue functions are a commonly used application of derivatives in economics. Revenue refers to the total amount of money a company earns from selling its products or services.

There are two types of revenue functions: total revenue (TR) and marginal revenue (MR).

## Total Revenue (TR):

Total revenue is the overall income generated by selling a certain quantity of products at a given price. It is calculated by multiplying the price (P) by the quantity sold (Q).

TR = P \* Q

For example, let's say a company sells widgets at a price of $5 each, and they sell 100 widgets. The total revenue (TR) would be:

TR = $5 \* 100 = $500

So, the total revenue generated is $500.

## Marginal Revenue (MR):

Marginal revenue is the additional revenue gained by selling one additional unit of a product. It represents the rate of change of total revenue with respect to the quantity sold. In other words, it is the derivative of the total revenue function with respect to quantity (dTR/dQ).

For example, suppose the total revenue function is given by:

TR = 100Q - 2Q^2

To find the marginal revenue function, we differentiate the total revenue function with respect to quantity (Q):

MR = dTR/dQ = 100 - 4Q

Now, let's evaluate the marginal revenue at a specific quantity. Suppose we want to find the marginal revenue when 50 units are sold. We substitute Q = 50 into the marginal revenue function:

MR = 100 - 4(50)

MR = 100 - 200

MR = -100

So, when 50 units are sold, the marginal revenue is -100.

A negative marginal revenue indicates a decrease in total revenue when one additional unit is sold. This occurs when the demand is elastic and the company needs to lower the price to sell the extra unit.

Understanding revenue functions and their derivatives, such as total revenue and marginal revenue, helps businesses make decisions regarding pricing strategies, production levels, and revenue optimization.

# Cost Functions

Derivatives are often used to analyze and optimize cost functions. Let's explore cost functions and their applications with an example.

## Total Cost (TC):

Total cost (TC) represents the overall expense incurred by a company in producing a certain quantity of products. It is the sum of all the costs, including fixed costs and variable costs, associated with production.

For example, consider a company that produces widgets. The total cost (TC) of producing Q widgets can be represented by the cost function:

TC = 1000 + 5Q + 0.1Q^2

In this cost function, 1000 represents the fixed costs, 5Q represents the variable costs, and 0.1Q^2 represents the additional costs that increase with the square of the quantity produced.

## Marginal Cost (MC):

Marginal cost (MC) measures the change in total cost resulting from producing one additional unit of a product. It is the derivative of the total cost function with respect to quantity (Q), or dTC/dQ.

To find the marginal cost function, we differentiate the total cost function with respect to quantity (Q):

MC = dTC/dQ = 5 + 0.2Q

Now, let's evaluate the marginal cost at a specific quantity. Suppose we want to find the marginal cost when 50 units are produced. We substitute Q = 50 into the marginal cost function:

MC = 5 + 0.2(50)

MC = 5 + 10

MC = 15

So, when 50 units are produced, the marginal cost is 15.

The marginal cost represents the additional cost incurred by producing one more unit. In this example, when 50 units are produced, the marginal cost is $15. This means that producing the 51st widget would cost an additional $15.

Cost functions and their derivatives play a vital role in production decisions and pricing strategies. Businesses use this information to optimize production levels and determine the appropriate pricing to maximize profits or minimize losses.